

Transportation Network Risk and Disruption



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Note to teachers: Solutions to exercises appear in red in the module.

Summary of Module

This module introduces the reader to the vulnerability analysis of a transportation system by modeling it using graphs or networks. Basic definitions and concepts from graph theory are introduced. Measures of node centrality are defined and illustrated. Using these concepts, hypothetical transportation systems are modeled and analyzed for their vulnerability to a failure or an attack. In the event of the disruption of a station or pathway, the consequences are quantified in terms of an increase in travel time or a reduction in the network capacity.

Target Audience

This module is written for undergraduate students in a freshman/sophomore level course in mathematics, engineering or environmental science.

Prerequisites

Students should be prepared for college level mathematics.

Topics

The topics in this module include graphs, transportation networks, measures of centrality, modeling transportation disruption, cost and loss of capacity.

Goals

Our goal is to introduce the student with minimal background to modeling a threat to a transportation network.

Anticipated Number of Meetings

2-3 Class periods.

1 class period for covering background material for graphs and measures for centrality.

1 class period for modeling transportation disruptions.

1 class period for further researching a topic of interest or constructing a model for a local transportation network.

Learning Outcomes

Students completing this module will be able to:

- Give definitions for and illustrations of basic graph theory concepts including: graph, node, edge, order, size, degree, neighborhood, path, distance, geodesic, bridge and cut-node.
- Interpret a transportation system modeled as a graph.
- State and apply the definitions of measures of centrality of nodes such as degree centrality and closeness centrality.
- Apply the centrality measures to a transportation network modeled as a graph.
- Analyze a disruption to a transportation network modeled by a graph.

Section 1.0 Introduction

Transportation systems are the lifeblood of modern society, enabling the movement of people and goods, supporting economic growth and connecting communities. From road networks and railways to airports, seaports, and public transit systems, these infrastructures are essential for our daily lives. However, they are also susceptible to a wide range of vulnerabilities that can disrupt their functionality, leading to significant social, economic and environmental consequences. Businesses rely on transportation networks to move their products and services, and delays or interruptions can lead to financial losses. Individuals rely on them to access education, healthcare, employment and social services. Disruptions to this access can disproportionately affect vulnerable populations. Since transportation networks can have adverse environmental impacts, including emissions, habitat disruption and pollution, analyzing their vulnerabilities can help engineers design more sustainable and eco-friendly networks. Finally, transportation networks are considered critical infrastructure. Their vulnerability can be exploited by those with malicious intent. Ensuring their security is a matter of national importance. See [7] for a thorough discussion of transportation disruptions and their impacts.

This module is designed to provide an introductory understanding of how the effects of a transportation network disruption can be modeled. Various factors make transportation systems susceptible to disruptions. Some examples of such factors are having a station or intersection located in a flood prone area or not having a robust way of connecting highly utilized stations or business districts within a city. To illustrate, in recent years there have been several bridge closures on major highways and interstates. One such closure was the I-40 bridge across the Mississippi River connecting West Memphis, AR to Memphis, TN on May 11, 2021. As reported in Kennedy [4] the closure was due to a fracture in a girder discovered during an inspection. The closure lasted almost four months. The total repair cost was \$10 million.

According to the Tennessee Department of Transportation, more than 40,000 vehicles cross the I-40 bridge daily, and about a quarter of them are trucks. The shutdown frustrated drivers, but also disrupted a major way of delivering goods across the country. Hence, the consequences of having these disruptions may be severe. The Mississippi River bridge example demonstrates that when a disruption does occur, there is a cost associated with the inefficiency that results. More often than major disruptions such as a bridge closure, we experience local disruptions due to accidents, weather, maintenance, rail station flooding or security issues, and others. We introduce the reader to how networks are modeled with graphs and how costs or inefficiencies from disruptions can be analyzed. By analyzing the characteristics of these networks, vulnerabilities, such as a network becoming disconnected, can be identified. Only by identifying and studying these vulnerabilities can risk managers begin to make the necessary recommendations to policy makers in order to create a more resilient system.

Exercise 1. Find and research a local transportation disruption that has occurred in your region over the past several years. Describe the transportation network affected by the disruption. What caused the disruption? Describe the consequences and cost of the disruption. Did it have economic impacts? Did it have environmental impacts or affect vulnerable populations? How might this disruption have been avoided? Have the causes in this case been addressed and the transportation network been made more resilient?

Section 2.0 Background material from Graph Theory

We begin by defining and discussing the basic ideas from graph theory that will be necessary to model our transportation network.

Definition. A *graph* G is a finite nonempty set V of objects called nodes (vertices), and a set E of two element subsets of V called edges.

Definition. $V(G)$ is the node set of G and $E(G)$ is the edge set of G . The *order* of G is the number of nodes in $V(G)$.

The graph G in Figure 1 has order $n = 7$, node set $V(G) = \{a, b, c, d, e, f, g\}$ and the edge set $E(G) = \{\{a, b\}, \{b, d\}, \{b, c\}, \{d, e\}, \{c, f\}, \{e, f\}, \{f, g\}\}$. We have labeled the edges on the graph to simplify the presentation, $\{a, b\} \rightarrow e_1, \{b, d\} \rightarrow e_2$, etc.

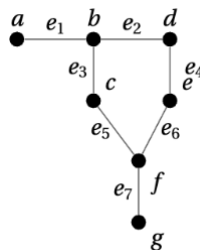


Figure 1. Graph G

Graphs can be used to model a wide range of real-world relationships, such as social networks, transportation systems, computer networks and more. We will use graphs to represent transportation networks. In a highway system, a node may represent a city, and the edge connecting two nodes may represent a highway. In a rail or subway network, the nodes may represent stations and the edges may represent rails and other infrastructure connecting the stations. For a bus network, the nodes may represent stops and the edges may represent a pathway of connected streets between stops. To further describe a transportation network, we need to know more about nodes and edges, their characteristics and the graph theory vocabulary used to describe them.

Definition. The *degree* of a node v in a graph G is the number of edges that are incident to it and is denoted $\deg v$.

Definition. Two nodes with an edge connecting them are *adjacent* nodes or neighbors. Also a neighborhood of a node is the set of all of its neighbors, denoted by $N(v)$ for a node v .

In Figure 1, node b and node f both have degree 3, whereas nodes c , d , and e each have degree 2. The neighborhood of node f , $N(f) = \{c, e, g\}$. One relationship that should be apparent is that $\deg v$ is the number of nodes in $N(v)$.

We need a few more terms to describe the topology of our networks

Definition. A *path* is a sequence of nodes where each adjacent pair is connected by an edge. A path is generally agreed to have no repeated nodes, except possibly the first node. The *length* of a path is the number of edges in it.

Definition. A *cycle* is a path that starts and ends at the same node. A graph containing no cycles is called a *tree*.

Definition. A graph is said to be *connected* if there is a path between any pair of nodes. If a graph is not connected, it may consist of multiple connected components.

In Figure 1, $P = \{a, b, c, f, e\}$ is a path of length 4, and $C = \{f, e, d, b, c, f\}$ is a cycle.

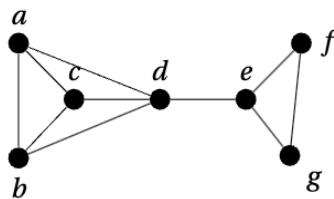


Figure 2. Graph H

Exercise 2. The graph H is given in Figure 2. Find each of the following for graph H.

1. The degree of nodes a , d , and f .
2. All of the cycles of graph H
3. Find the path of shortest length connecting $\{a, g\}$, $\{c, f\}$ and $\{b, e\}$.
4. The neighborhoods $N(c)$, $N(f)$, and $N(d)$.

Solution.

1. $\deg a = 3$, $\deg d = 4$, $\deg f = 2$.
2. $C_1 = \{a, b, c, a\}$, $C_2 = \{a, b, d, a\}$, $C_3 = \{a, c, d, a\}$, $C_4 = \{a, c, b, d, a\}$, $C_5 = \{b, d, c, b\}$, $C_6 = \{e, f, g, e\}$.
3. Shortest length connecting $\{a, g\}$: $\{a, d, e, g\}$, $\{c, f\}$: $\{c, d, e, f\}$
 $\{b, e\}$: $\{b, d, e, \}$
4. $N(c) = [a, b, d]$, $N(f) = \{e, g\}$, and $N(d) = \{a, b, c, e\}$.

Section 3.0 Measures of Centrality

Centrality measures are often used as tools in the analysis of networks. They help identify and quantify the importance of individual nodes or edges. To a certain extent, it is a measure of central tendency that can provide insights into the network's structure. The centrality measures are defined in such a way that the node or edge assigned the higher number is more central than the node or edge assigned a lower number.

Centrality in transportation networks helps us to assess the importance or influence of specific network elements, such as nodes (e.g., intersections, bus stops, airports) or edges (e.g., roads, rail tracks, flight routes). The central elements are those that play a critical role in facilitating the flow of goods, people or information within the network. One might expect a city's business district to be close to hubs of high centrality. Also, a disruption near a place of high centrality may be more impactful and cause a higher cost to the community than one at a lower centrality, Ghazaryan [6].

There are several centrality measures commonly used in transportation network analysis. We will define and discuss two, degree centrality and closeness centrality. Degree centrality measures the number of connections (edges) a node has in the network.

Definition. The *degree centrality* of a node v in a graph G of order $n > 2$ is

$$C_d(v) = \frac{\deg(v)}{n-1}. \quad (1)$$

In a social network, an influencer has a large number of followers and hence can be identified as having a high degree centrality. In transportation networks, nodes with high degree centrality represent critical junctions or hubs, such as major intersections or transportation terminals. It is a

simple and intuitive measure but may not capture the importance of nodes in facilitating global network connectivity.

Closeness centrality measures how quickly a node can reach all other nodes in the network. In transportation networks, nodes with high closeness centrality are essential for reducing travel times and improving accessibility. Airports or transit centers with high closeness centrality enhance connectivity within a region. To define closeness, we need a measure of average closeness between nodes.

Definition. Let u and v be nodes in a connected graph G . Then

- a. a $u - v$ geodesic is a $u - v$ path of minimum length;
- b. the distance $d(u, v)$ between u and v is the number of edges in any $u - v$ geodesic.

The average distance between node v and the remaining nodes in $V(G)$ can then be calculated,

$$\frac{\sum_{u \in V(G)} d(v, u)}{n-1} \quad (2)$$

and define closeness centrality to be its reciprocal as given in (3). Since the average distance is greater than one, $C_c(v) < 1$ and the smaller the average distance the larger the value of $C_c(v)$ indicating greater closeness centrality.

Definition. The *closeness centrality* of a node v in a graph G of order $n > 2$ is

$$C_c(v) = \frac{n-1}{\sum_{u \in V(G)} d(v, u)}. \quad (3)$$

Consider Figure 2, a standard figure used to illustrate centrality measures in texts and papers.

Example 1. Calculate the degree and closeness centrality for node a in Graph H shown in Figure 2.

Solution. Since the order of the graph H is $n = 7$, and $\deg(a) = 3$, then $C_d(a) = 3/6 = 1/2$. To compute the closeness centrality of node a , we first identify a geodesic with node a and every other node and then calculate its distance. It is easy to see $d(a, b) = d(a, c) = d(a, d) = 1$ since the geodesics are single edges. Path $\{a, d, e\}$ is an $a - e$ path with just two edges (minimum length) and hence a geodesic, $d(a, e) = 2$. Likewise, the reader can verify $d(a, f) = d(a, g) = 3$. Therefore,

$$C_c(a) = \frac{6}{1+1+1+2+3+3} = 6/11. \quad \blacklozenge$$

Solving Exercise 3, the reader will show that node d is the most central node in Graph H. The reader will verify $C_d(d) = 0.67$ and $C_c(d) = 0.75$. Node d also has the property that if removed, the graph becomes disconnected and is separated into two *component* graphs. The same can be said for removing edge $\{d, e\}$. This characteristic is obviously important in

analyzing the disruption of a transportation network and can be summarized in the following definitions.

Definition. In a disconnected graph, each separate connected subgraph of G is a *component* of G .

Definition. A *bridge* is an edge of a graph whose removal increases the number of components of a graph.

Definition. A *cut-node* of a graph G is a node whose deletion increases the number of components of the graph.

In summary, centrality measures play a vital role in analyzing and managing transportation networks. They help identify the most critical elements and guide decision-making processes to improve network efficiency, safety and resilience. Understanding centrality measures helps optimize traffic signal timing, manage congestion and implement rerouting strategies during incidents. Identifying nodes with high centrality is crucial for efficient emergency response planning and ensuring rapid access to critical locations. Centrality measures can aid in optimizing public transportation routes, determining the locations of key transit hubs and enhancing the overall transportation system. Identifying central nodes can help optimize the logistics of moving goods, minimizing transportation costs and improving supply chain efficiency. Finally, assessing centrality can reveal critical nodes vulnerable to both natural and targeted disruptions, guiding resilience strategies and disaster recovery planning.

The choice of centrality measure should align with the specific goals and characteristics of the transportation network under analysis. We have only demonstrated two measures of centrality in this section, but there are other measures that the reader may wish to investigate such as betweenness and eigenvector centrality. For example, see either Cockburn [2] or Watson [5] for a very accessible discussion and examples of betweenness and eigenvector centrality.

Exercise 3. Compute the degree and closeness centrality of each node in Graph H shown in Figure 2.

Solution.

node	a	b	c	d	e	f	g
$\deg v$	3	3	3	4	3	2	2
$C_d(v)$	0.5	0.5	0.5	0.67	0.5	0.33	0.33
$C_c(v)$	0.54	0.54	0.54	0.75	0.67	0.46	0.46

Exercise 4. Compute the degree and closeness centrality of each node in Graph J shown in Figure 3.

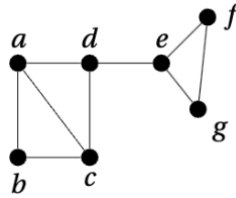


Figure 3. Graph J

Solution.

node	a	b	c	d	e	f	g
deg v	3	2	3	3	3	2	2
$C_d(v)$	0.5	0.33	0.5	0.5	0.5	0.33	0.33
$C_c(v)$	0.55	0.4	0.55	0.67	0.6	0.43	0.43

Section 4.0 Modeling a Transportation Disruption

Cost and efficiency normally play an important role in analyzing a transportation network. If the network is disrupted in some way, then the disruption can be measured in a loss of efficiency or possibly an increase in transportation cost. We have likely all experienced a disruption in our travel due to an accident or construction on a highway. Xie [1] provides the example of the collapse of the I-35 Mississippi River bridge in Minneapolis, MN in 2007. This collapse cut a major commuting route to downtown Minneapolis costing an estimated \$400,000 to reroute traffic for just one day. The I-35 bridge collapse was due to a design flaw. However, many transportation disruptions happen due to natural causes. A subway terminal below street level, and possibly below sea-level, is subject to the risk of a weather-related flood event or even future flooding due to climate change. In this section we demonstrate models that allow us to analyze this type of transportation risk.

First, let's consider the cost. With our edges labeled, $E(G) = \{e_1, \dots, e_n\}$, we can associate with each edge a cost, $\{c_1, \dots, c_n\}$. This association can be made explicit in the pair $\{e_i, c_i\}$. This cost may vary with time or be constant. For example, one component of interest for cost in a transportation network is travel time. Travel time may be higher during the 7am-9am and 4pm-6pm commuting times. However, for our purposes, let's consider it to be an average commuting time for a given duration. We summarize the analysis in Example 2 where the travel time is indicated by edge.

Example 2. Consider Figures 4 and 5. Edge travel time, t_i , for edge e_i is shown in the pair $\{e_i, t_i\}$ used to label each edge. We calculate the time to travel from one node to another by summing the travel times in the path taken. To travel from node d to node g by the path

$\{d, e, f, g\}$ takes a travel time of $4 + 7 + 4 = 15$. If either edge e_4 or edge e_6 is eliminated or node e removed, shown in Figure 5, then the traveler would be forced to take path $\{d, b, c, f, g\}$ resulting in a travel time of 16. Hence, either of these three disruptions results in an increase in travel time of 1. \blacklozenge

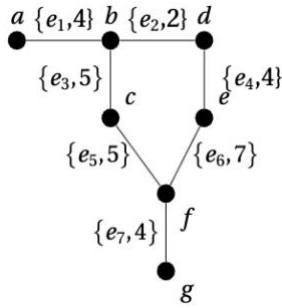


Figure 4. Edge Time

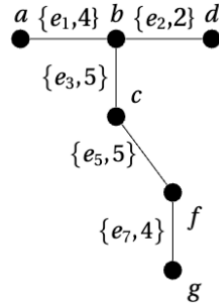


Figure 5. Node e Removed

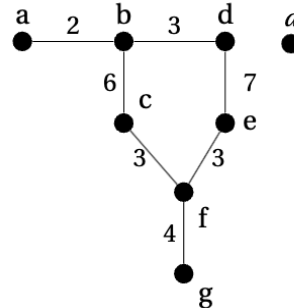


Figure 6. Edge Capacities

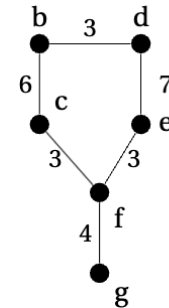


Figure 7. Edge Removed

If the network represents a rail or subway system, then utilization rate or capacity may be a better choice for the measurement of interest, although time spent at the station may also be relevant. A more thorough and technical discussion of capacity for transportation networks can be found in Yang [3]. In our case, the capacity may be the average number of persons the particular link (represented by the edge) can accommodate in a specified time duration. For illustration, in Figure 6 we have labeled each edge with its capacity. We consider a disruption resulting in the loss of service within the network. This loss may be due to the loss of service at a node or along a path. We will denote the capacity for edge e_k using the pair $\{e_k, c_k\}$ in labeling the graph. When the edges are known, we simplify the labeling by just using the value c_k as shown in Figures 6 and 7. Let the normal capacity for an edge e be denoted $Cap(e)$ and the capacity for the edge after a disruption be denoted by $Cap_d(e)$. In a transportation network that is represented by a graph of order n , the total capacity of a network that has not been disrupted is just the sum of the edge capacities,

$$TC = \sum_{e \in E(G)} Cap(e). \quad (4)$$

After a disruption to the network, we recalculate the capacity for each edge so that $Cap_d(e) = 0$ if the edge has been totally disabled or some fraction of the normal capacity if the link has been impaired due to the disruption. We illustrate these definitions and calculations in Example 3.

Example 3. Consider the graph with edge capacities illustrated in Figure 6. We have a total capacity in this network of

$$TC = 2 + 3 + 6 + 7 + 3 + 3 + 4 = 28.$$

In Figure 7, edge e_1 has been removed resulting in the isolation of node a , and hence a loss in capacity of 2 since $Cap_d(e_1) = 0$. So, the total capacity after the disruption is $TC_d = 26$.

Therefore, after the disruption the network capacity was decreased by $1 - 26/28 = 0.071$ or 7.1%. ◆

The calculation in Example 3 can be carried out again and again as further disruptions occur and repairs are made. However, if a disruption causes the network to become disconnected into component subgraphs, then clearly something has happened to cause a loss of network efficiency not measured by the sum of the edge capacities. Not being able to get from one part of the network to another part within the network needs to be measured. To model this loss of efficiency, we will adopt a model from power networks that measures a loss of connection when a power network is disrupted, see Cockburn [2]. First define

$$\beta(v) = N_g(v)/TC_d \tag{5}$$

where $N_g(v)$ is the capacity of the largest connected subgraph containing v after the disruption. Then define $\hat{Cap}_d(e) = \beta(v) \cdot Cap_d(e)$ for each edge adjacent to v . Notice $\hat{Cap}_d(e)$ is well defined. If an edge is adjacent to both nodes u and v , then u and v are necessarily in the same connected subgraph and $\beta(u) = \beta(v)$. We will call $\hat{Cap}_d(e)$ the *efficiency capacity* of edge e after the disruption. Observe that $\beta(v)$ is just a proxy for the percentage of individuals in the network that can utilize the capacity of those edges. If the network remains connected after the disruption, then $N_g(v) = TC_d$ so that $\beta(v) = 1$ for each remaining node v and the efficiency capacity is just equal to the edge capacity. In this case, the decrease in network efficiency is just the decrease in network capacity. However, if the network becomes disconnected, the decrease in network efficiency is given by

$$1 - \frac{1}{TC} (\hat{Cap}_d(e_1) + \hat{Cap}_d(e_2) + \dots + \hat{Cap}_d(e_n)). \tag{6}$$

We illustrate in Example 4.

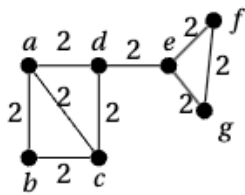


Figure 8. Graph K

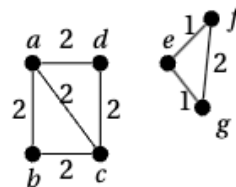


Figure 9. Bridge $\{d, e\}$ removed

Example 4. Consider the network modeled by Graph K shown in Figure 8. Take $Cap(e_k) = 2$ for all nine edges. We have $TC = 18$. Suppose the bridge, edge $\{d, e\}$ is removed, shown in Figure 9. Then $\hat{Cap}_d(\{d, e\}) = 0$. Also, we see in Figure 9 that the cause of the disruption damaged the station at node e , causing a loss in capacity to the connecting links (adjacent edges $\{e, f\}$ and $\{e, g\}$). The resulting total capacity of the network after the disruption is $TC_d = 14$. However, the disruption has also disconnected the network into two components. Let's compute

the loss of network efficiency. $\beta(v) = 5/7$ for nodes $a, b, c,$ and d . $\beta(v) = 2/7$ for nodes $e, f,$ and g . Hence,

$$1 - \frac{1}{TC} (\hat{C}ap_a(v_1) + \dots + \hat{C}ap_d(v_n)) = 1 - \frac{1}{18} \left(\frac{5}{7}(2 + 2 + 2 + 2 + 2) + \frac{2}{7}(1 + 1 + 2) \right) = 0.54. \quad \blacklozenge$$

Therefore, the disruption causing the link represented by edge $\{d, e\}$ to be disabled and damaging the station at node e , resulted in a loss of network capacity by $4/18$ or 22.2% , but the network efficiency was reduced by 54% , reflecting the fact that $5/7$ of the network capacity was cut-off from the other $2/7$ of the network. If further disruptions occur, the calculation can be made again with each disruption showing a further loss in capacity and efficiency.

Watson [5], builds a methodological framework for simulating failure and recovery of rail networks under compound natural and opportunistic hazards. The idea behind the compound hazard is that a natural hazard, such as flooding, causes a disruption or failure in the network. In its weakened, more vulnerable state, there is an increased likelihood of an opportunistic enemy attacking the same network before recovery begins. In their development of this framework, Watson [5] uses centrality to help identify the potential targets of an opportunistic attack, where the components of greatest centrality become the object of the strike. We illustrate this idea through Exercise 5.

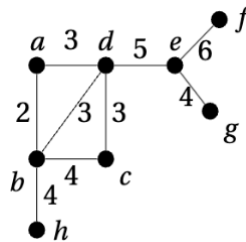


Figure 8. Graph M

Exercise 5. Graph M in Figure 8 models a light rail system with a station (node c) and rail (edge $\{e, f\}$) both in flood prone areas. An engineer is studying the risk posed by a 100-year flood event followed by an opportunistic targeted attack. In the event of a 100-year flood, assume the station at node c and its adjacent rails are totally disabled and the capacity of the rail represented by edge $\{e, f\}$ is reduced by 50% .

- Calculate the capacity of the network before the flood.
- After the 100-year flood event, recalculate the capacity of the network.
- What is the percentage loss in capacity and the percentage loss in network efficiency?
- Calculate degree and closeness centrality of each node remaining after the flood event.
- The engineer assumes a targeted attack will occur immediately after the flood event at the node with the highest closeness centrality. This attack will remove the adjacent rail with the highest capacity and leave the other adjacent rails unaffected. Recalculate the loss in network capacity and loss of network efficiency.

Solution.

A. $TC = 3 + 2 + 4 + 3 + 3 + 4 + 5 + 6 + 4 = 34$, see Figure 8.

B. $TC_d = 3 + 2 + 0 + 0 + 3 + 4 + 5 + 3 + 4 = 24$, see Figure 9.

C. Network capacity and network efficiency are reduced by 29%. $\beta(v)/24 = 1$ for each node since the model graph is still connected after the flood: $1 - 24/34 = 0.29$.

D.

node	<i>a</i>	<i>b</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>
deg <i>v</i>	2	3	3	3	1	1	1
$C_d(v)$	0.33	0.5	0.5	0.5	0.17	0.17	0.17
$C_c(v)$	0.50	.55	.67	.60	.40	.40	.375

E. The nodes with the largest centrality measures are $C_d(b) = C_d(d) = C_d(e) = 0.5$ and $C_c(d) = 0.67$. Therefore, the targeted attack will take place at node *d*, disabling edge {*d,e*} since it is the adjacent edge with the highest capacity, Figure 10.

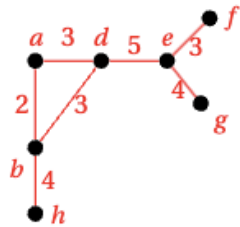


Figure 9: After the flood event

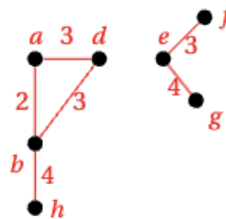


Figure 10: After the targeted attack

$TC_{Attack} = 19$. For nodes *a*, *b*, *d*, and *h*, $\beta(v)/19 = 12/19$ and for nodes *e*, *f*, and *g*, $\beta(v)/19 = 7/19$. After the two disruptions, the reduction in network capacity is $1 - 19/34 = 0.44$. The network efficiency reduction is 70%,

$$1 - \frac{1}{34} \left(\frac{12}{19} (2 + 3 + 3 + 4) + \frac{7}{19} (3 + 4) \right) = 0.70.$$

Section 5.0 Further work or projects

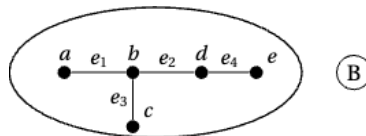


Figure 11. Graph simplification

A graph modeling a transportation network may be further simplified. For example, connected nodes in a subgraph may be combined. In the subgraph shown in Figure 11, {*a*, *b*, *c*, *d*, *e*}, the nodes and edges circled may be combined into a single node *B*. In this case, if the edges are assigned a specific value for capacity, then the total capacity of the edges included in the new node is assigned to the new node. For example, in Figure 11, if $Cap(e_1) = \dots = Cap(e_4) = 3$,

then the new node B is assigned a capacity of 12.

Project. Identify a local transportation network of interest. Model the network with a graph. You may wish to use a simplified model with certain subgraphs combined into nodes. Calculate node centrality measurements. What is the importance of the more central nodes? Estimate the edge capacities with higher usage links receiving a higher edge capacity measure. Analyze the risk of disruption by experimenting with node and edge removals. Use travel time if it is of more interest than capacity. In this scenario, alternate routes must be considered in case of a disruption.

References

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